

This **JOS Plus Only** article is an expanded version of the article of the same title appearing in JOS 20:1 (Spring, 2011). This **JOS Plus Only** article contains 9 pages of supplemental information added to the JOS version.

The Chinese JIANGSHI Slide Rule

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The Chinese JIANGSHI type YI (B) Slide Rule is probably the most unusual of all of those shown in my JOS listing of slide rules with hyperbolic function scales.¹ It has a very different design than the other slide rules. There are numerous odd scales and marks. Also, it varies in the manner by which the hyperbolic function values are obtained.

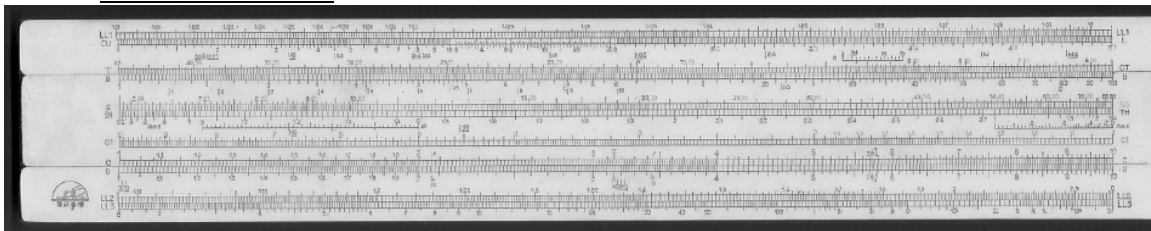
The following translation describing this slide rule was obtained from a Chinese internet site:

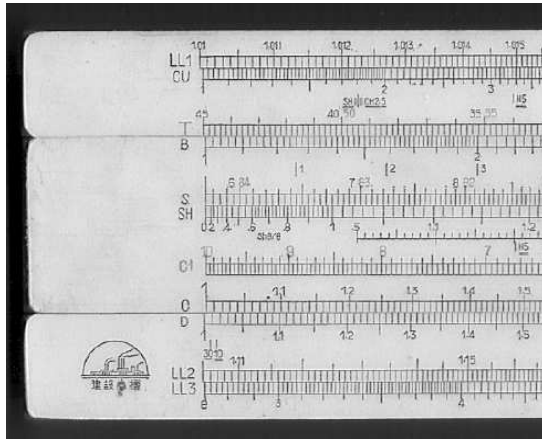
- “1. The one-sided Jiangjī’s type B slide rule is designed by Mr. Jiang Dequ, from JIN HUA YINGSHI University, in Zhe Jiang province.*
- 2. The surface of this slide rule was printed through photomechanical process and coated with transparent plastic then stuck on the wooden slide rule. Although it is not exquisite enough and scale is not very accurate, it was the first slide rule designed and made by Chinese people between 1937 and 1945. The structure of this slide rule is rather simple, it is also easy to make and it is sold at lower price. So it was very popular with users at that time. Even during early days of the P.R.C., it was also sold in Shanghai and Hangzhou”.*²

The actual date this slide rule was introduced and the years it was sold is not known. However, if it was first sold in the year 1937 it would have been third on the list of slide rules with hyperbolic function scales. The first was the K&E Log Log Duplex Vector 4093-3, introduced in late 1929. Second was the Hemmi 153 in 1933.

There are two slide rule variations and this is what they look like:

FIRST VERSION

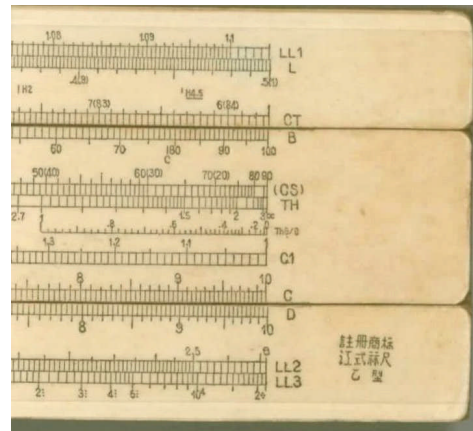
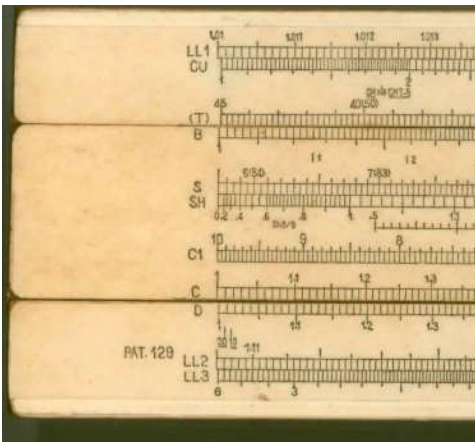
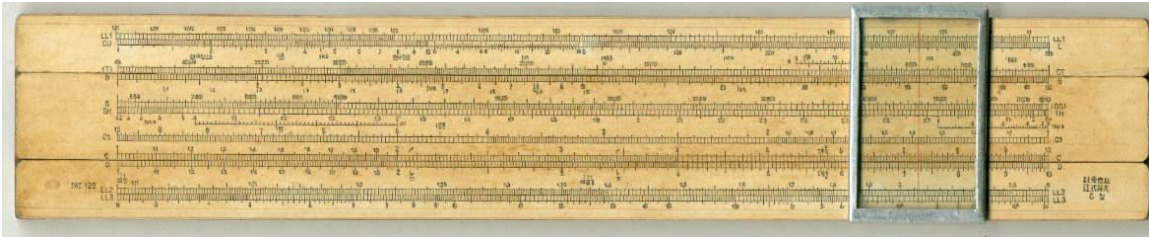




The only scales are on the front side. The back side of the first version has instructions on using the scales to solve problems, and to the right a *Weight of Metals* table is shown. The back side of this appears below:

1/10	0.0625	1/16	0.03125	Base of hyp. log.	Scale C	Scale D	C	D	Scale C	Scale D	C	D	Scale C	Scale D	C	D	Weight of Metals	
1/8	0.1250	1/8	0.1250	2.30103	Circle	1	2	Sq. ft.	Sq. meters	140	15	100	Scale C <td>Scale D <td>C <td>D</td> <td>lb./cu.in.</td> <td>kg./cu.cm.</td> </td></td>	Scale D <td>C <td>D</td> <td>lb./cu.in.</td> <td>kg./cu.cm.</td> </td>	C <td>D</td> <td>lb./cu.in.</td> <td>kg./cu.cm.</td>	D	lb./cu.in.	kg./cu.cm.
3/16	0.1875	1/4	0.2500	2.40483	Circum. "	10	3	Yards	Cub. m.	61	51	100	Scale C <td>Scale D <td>C <td>D</td> <td>Feet of water</td> <td>Atmosphere</td> </td></td>	Scale D <td>C <td>D</td> <td>Feet of water</td> <td>Atmosphere</td> </td>	C <td>D</td> <td>Feet of water</td> <td>Atmosphere</td>	D	Feet of water	Atmosphere
1/4	0.2500	1/4	0.2500	2.50000	Side, inscribed Sq.	100	20	Feet	Cub. ft.	36	500	100	Scale C <td>Scale D <td>C <td>D</td> <td>Wron's Iron "</td> <td>Cast "</td> </td></td>	Scale D <td>C <td>D</td> <td>Wron's Iron "</td> <td>Cast "</td> </td>	C <td>D</td> <td>Wron's Iron "</td> <td>Cast "</td>	D	Wron's Iron "	Cast "
5/16	0.3125	1/2	0.5000	2.69897	" " equal Sq.	100	20	Feet	Imp. gallons	2000	85	100	Scale C <td>Scale D <td>C <td>D</td> <td>Steel</td> <td>Copper</td> </td></td>	Scale D <td>C <td>D</td> <td>Steel</td> <td>Copper</td> </td>	C <td>D</td> <td>Steel</td> <td>Copper</td>	D	Steel	Copper
3/8	0.3750	3/8	0.3750	2.87549	Diagonal of Sq.	70	99	" "	U.S. "	17	100	100	Scale C <td>Scale D <td>C <td>D</td> <td>Brass</td> <td>Lead</td> </td></td>	Scale D <td>C <td>D</td> <td>Brass</td> <td>Lead</td> </td>	C <td>D</td> <td>Brass</td> <td>Lead</td>	D	Brass	Lead
7/16	0.4375	1/2	0.5000	3.09104	Area, circle "	70	99	" "	Liters	6100	22	100	Scale C <td>Scale D <td>C <td>D</td> <td>Zinc</td> <td>Aluminum</td> </td></td>	Scale D <td>C <td>D</td> <td>Zinc</td> <td>Aluminum</td> </td>	C <td>D</td> <td>Zinc</td> <td>Aluminum</td>	D	Zinc	Aluminum
1/2	0.5000	1/2	0.5000	3.32193	Ann. inscribed Sq.	2	94	" "	French R.P.724	6700	29	100	Scale C <td>Scale D <td>C <td>D</td> <td></td> <td></td> </td></td>	Scale D <td>C <td>D</td> <td></td> <td></td> </td>	C <td>D</td> <td></td> <td></td>	D		
5/8	0.6250	3/4	0.7500	3.57978	Millimeters	2	94	" "	U.S. "	214	1600	100	Scale C <td>Scale D <td>C <td>D</td> <td></td> <td></td> </td></td>	Scale D <td>C <td>D</td> <td></td> <td></td> </td>	C <td>D</td> <td></td> <td></td>	D		
3/4	0.7500	3/4	0.7500	3.87603	Meters	200	61	" "	kg.	750	340	100	Scale C <td>Scale D <td>C <td>D</td> <td></td> <td></td> </td></td>	Scale D <td>C <td>D</td> <td></td> <td></td> </td>	C <td>D</td> <td></td> <td></td>	D		
7/8	0.8750	1	1.0000	4.19079	Miles	5	32	" "	French R.P.724	6200	38	100	Scale C <td>Scale D <td>C <td>D</td> <td></td> <td></td> </td></td>	Scale D <td>C <td>D</td> <td></td> <td></td> </td>	C <td>D</td> <td></td> <td></td>	D		
1	1.0000	1	1.0000	4.54242	Sq. cm.	1	100	Pou. (long)					Scale C <td>Scale D <td>C <td>D</td> <td></td> <td></td> </td></td>	Scale D <td>C <td>D</td> <td></td> <td></td> </td>	C <td>D</td> <td></td> <td></td>	D		

SECOND VERSION



The back side of the second version is beautifully clear lacquered and has three screws to adjust the scale alignments and the tension on the middle slide part. It is believed that as the first version of the rule was used it was found that a slide adjustment was needed. The

screws were then added to this second version as shown below. There are no printed items on the back of this second slide rule.



Richard Smith Hughes owns the first version, and sent the above pictures of it to me. The scales seem to be the same on the two versions, but the scale lettering and the logos on the lower stock of each are different.

On the first version the major scales T and CS are not shown with parentheses as in the second version, i.e. (T) and (CS). Also, these letters T and CS are in Red color, and the numbers 50, 60, etc., are in Red with no parentheses. The scale letter L, on the right side, is in black, but its numbers .6, .7, etc., are in Red with no parentheses. The scale letter T, on the left side, and its numbers 50, 55, etc., are in Red, all with no parentheses. The scale letters CS, on the right side, and its numbers 84, 83, etc., are in Red, all with no parentheses. The scale letters CI, right and left side, and the scale numbers are Red all of the way. The mark π is in Red on scales B, C, and D. There is no π on these scales in the second version slide rule. There is no Red on the second version slide rule. It is in black print only.

On the first version there is a picture in the lower left hand corner of a factory with smoke stacks and Chinese characters underneath; and, there is no logo on the bottom right side of this first slide rule. The second slide rule version has the words "PAT 129" in the lower left hand corner; and, in the lower right hand corner are three lines of Chinese writing.

The cursors are different. The first version one is plastic and not metal framed. It has two hairlines that cover the B scale. The hairline on the left gives the measure $\pi/4$. The hairline on the right gives about the number 142. Its usage is unknown. The cursor on the second slide rule version is glass and is framed in metal. It has no hairlines on the glass.

The width of the first version is 49 cm and the second is 54 cm. Length is the same on each. Remember that the scales are printed on strips of paper and glued on. The strips are cut the same width for each rule. The reason for the difference in width between the rules is not known. Perhaps the slide rules were manufactured in two different places.

On the case is a picture showing some factory buildings and Chinese characters. This is what it looks like:



There is a manual written in the Chinese language. The numerical explanation of some of the slide rule functions were taken from this manual for this Article.

THE MAJOR SCALES, SUB-SCALES AND GAUGE MARKS

It is obvious that this slide rule was designed for use by an Electrical Engineer. This is seen from the following listing of the various scales. This list is copied from the Manual. Some of these are made up of multiple sub-scales on one line, and between the major scales are found many special scales and unusual gauge marks. All of the scales and ranges as they were listed in the manual are shown below:

<u>Name of Scale</u>	<u>Range of Scale</u>
LL1	1.01 – 1.105
Cu	1 – 10
L	0 – 0.5 (0.5 – 1.0)
O – 8	0 – 8 (8 – 16)
O – 30	0 – 30 (30 – 60)
SH, CH, H	H1 – H45
W	
R	0° - 75° °C
(T)	45° - 5.71° (84.29°)
CT	0.71° - 45°
B	1 – 1,000
1/3, 2/3	
CC1	
1 – 10	1 – 10
S	5.74° - 90°
(CS)	0° - 84.26°
SH	1 – 2.7
Sh θ / θ	0 – 1
TH	1 – ∞
Th θ / θ	0 – 1
ΔH	0 – 0.5 and 0.5 – 1.0
H5, H10	
C1	1 – 10
C	1 – 10
TS°, /	
D	1 – 10
30, 10, 40, 20, 0	
LL2	1.105 – e
LL3	e – 22,000

THE SCALES AND GAUGE MARKS

When looking at this list it seems the scales and gauge marks would best be presented in some functional order. Rather than hit and miss as listed above. We will use the following four general selections when discussing the scales and gauge marks:

- 1) Math: C, D, CI, CU, B, L, and LL1, 2, 3.
- 2) Trig: S, T, CT.
- 3) Elektro: Weight, Resistance/Temperature. Wire diameter, 30/10, 40/20/0, and 1 to 10 scales.
- 4) Hyperbolic: SH, CH, TH, Sh θ/θ , Th θ/θ , ΔH , and the H's.

This listing is in summary only and does not show all of the scales and gauge marks. However, all will be included in their proper area in this Article.

MATH SCALES AND GAUGE MARKS

- **LL1** on left and **LL1** on the right. This is a usual slide rule Log Log scale with range ascending from 1.01 to 1.105. It is used together with the C scale to find the n^{th} , or to find the $1/n^{\text{th}}$, power of a number.
- **LL2** on the left and **LL2** on the right. This is a usual LL2 scale with range ascending from 1.105 to e. It is used together with the C scale to find the n^{th} , or to find the $1/n^{\text{th}}$, power of a number.
- **LL3** on the left and **LL3** on the right. This is a usual LL3 scale with range ascending from e to 22,000. It is used together with the C scale to find the n^{th} , or to find the $1/n^{\text{th}}$, power of a number.
- **CU** on the left, and **L** on the right. This major scale is divided into four major sub-scales, as follows:

•(1). CU: The CU sub-scale is on the far left and runs from 1 to 10. It is used to find cube roots together with the C scale. Also used are the $1/3$ and $2/3$ marks found under scale B. Examples of cube root: $(6.4)^{1/3} = 1.856$, set cursor at 6.4 on CU and read 1.856 under cursor on D. Continuing with $(64)^{1/3} = 4$, and $(640)^{1/3} = 8.61$. Slide left index of C under the cursor. Then slide cursor to $1/3$ mark (under about 4.6 on the B scale) this will give $(64)^{1/3} = 4$ on the D scale. Slide cursor to $2/3$ mark (under about 21.5 on B scale) this will give $(640)^{1/3} = 8.61$ on the D scale.

Cube roots of the numbers from 1 to 10 fall in the range from 1 to 2.154. These are found by reading directly under the hairline from CU to the C scale. Cube roots of the numbers from 10 to 100 fall in the range from 2.154 to 4.642. These are found by bringing the $1/3$ mark found under the B scale to the left index of the CU scale. Then the cube roots of the numbers from 10 to 100 are found by reading directly under the hairline from CU to the C scale. Cube roots of the numbers from 100 to 1000 fall in the range from 4.642 to 10. These are found by bringing the $2/3$ mark found under the B scale to the left index of the CU scale. Then the cube roots of the numbers from 100 to 1000 are found by reading directly under the hairline from CU to the C scale. Cube roots of 1000 times or 1/1000 times the above ranges are found by recognizing that the cube root of 1000 is 10, and the cube root 1/1000 as 1/10.

•(2) and (3). There are two sub-scales in the middle that have no letter name. The first ranges from 0 to 8, and the second from 0 to 30. The use of the 0 to 8 scale is unknown except for obtaining the decimal value of fractions from $1/16^{\text{th}}$ to

15/16ths. The use of the 0 to 30 scale is to obtain the decimal equivalents of minutes (of degrees). They both are to be read using the scales directly above them. On inspection we find these upper scales are divided into 5 main segments, and each segment has sub divisions.

To find values using the 0 to 8 scale we need to make two arrangements of the bottom and top scales as follows:

0 0.1 0.2 0.3 0.4 0.5 and, 1.0 0.9 0.8 0.7 0.6 0.5
 0 4 8 16 12 8

From these arrangements the values of the 16ths from 1/16th to 15/16ths can be read on the top scale by setting the hairline to the various selected marks on the bottom scale.

To find values using the 0 to 30 scale we need to make two arrangements of the bottom and top scales as follows:

0 0.1 0.2 0.3 0.4 0.5 and, 1.0 0.9 0.8 0.7 0.6 0.5
 0 10 20 30 60 50 40 30

From these arrangements the values of the decimal equivalents of minutes (of degrees) can be read on the top scale by setting the hairline to the various selected marks on the bottom scale. Decimals are useful as the S and T scales are decimal.

▪(4). L: The L scale is found on the right side, and is used for finding logs to the base 10. First move the slide to the right until the number 10 on the B scale (in the middle) is aligned with the right index of the CT scale. The logs from 1 to 3.16 are found by setting the number on the C scale and reading the log under the hairline on the L scale. The logs from 3.16 to 10.00 are found by setting the number on the C scale and reading the log under the hairline on the L scale.

- **B** on the left and **B** on the right. This is the usual squares and square roots scale with the values read on scales C and D. Identical gauge marks are found in two places on scale B. They are found at $0.7854 = \pi/4$. It is used as an aid in finding the area of a circle by the formula $A = \pi D^2 / 4$.
- **C1** on the left and **C1** on the right. This is the usual slide rule CI reciprocal inverted scale.
- **C** on the left and **C** on the right. This is the usual slide rule C scale. It has three separate marks: Mark " at $206,265 = (180 \times 60 \times 60) / \pi$; Mark ' at $3,438 = (180 \times 60) / \pi$; and at $\underline{T} 1 | 1 \underline{S}^\circ$ we have the radian Mark $57.3^\circ = 180^\circ / \pi$. The sin and tan of small angles (less than 5.74°) may be found using this radian Mark (See Examples below under the Trig sin and tangent headings).
- **D** on the left and **D** on the right. This is the usual slide rule D scale. It has the same three marks as C, i. e. ", ', and $\underline{T} 1 | 1 \underline{S}^\circ$.

Some Additional Math Procedures

- To find $\text{Log}_e y = x$: Examples; (1) $\text{Log}_e 5.16 = 1.64$, set 5.16 on LL3 and read 1.64 on C. (2) $\text{Log}_e 1.58 = 0.425$, set 1.58 on LL2 and read 0.457 on C. (3) $\text{Log}_e 1.06 = 0.0583$, set 0.106 on
- To find $\text{Log}_{10} y = x$: Move the cursor to 10 on LL3 and pull the left index on C under the cursor, then proceed. Examples; (1) $\text{Log}_{10} 60.0 = 1.78$, set 60.0 on LL3 and read 1.78 on C. (2) $\text{Log}_{10} 1.72 = 0.235$, set 1.72 on LL2 and read 0.235 on C.
- To find $x^c = y$ using the left index on C: Move the cursor to 8 on LL3 and pull the left index on C under the cursor, then proceed. Examples; (1) $8^{1.6} = 27.8$, set 1.6 on C and read 27.8 on LL3. (2) $8^{0.16} = 1.395$, set 0.16 on C and read 1.395 on LL2. (3) $8^{0.016} = 1.0338$, set 0.016 on C and read 1.0338 on LL1.

- To find $x^c = y$ using the right index on C: Move the cursor to 1.92 on LL2 and pull the right index on C under the cursor, then proceed. Examples; (1) $1.92^{6.15} = 55.0$, set 6.15 on C and read 55.0 on LL3. (2) $1.92^{0.615} = 1.493$, set 0.615 on C and read 1.493 on LL2. (3) $1.92^{0.0615} = 1.0409$, set 0.0615 on C and read 1.0409 on LL1.

- To find $x^{1/c} = y$ using the left index on C: Move the cursor to 32 on LL3 and pull c on C under the cursor, then proceed. Examples; (1) $32^{1/2.5} = 4.0$, set 2.5 on C and read 4.0 on LL3 under the left index of C. (2) $32^{1/25} = 1.149$, set 25 on C and read 1.149 on LL2 under the left index of C. (3) $32^{1/250} = 1.014$, set 250 on C and read 1.014 on LL1 under the left index of C. (Note: You only have to set the slide rule one time).

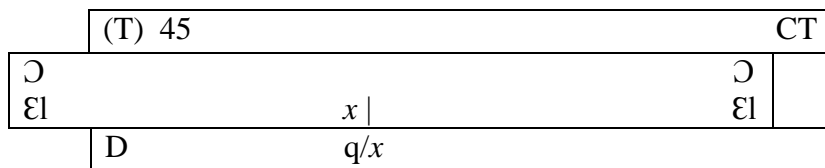
- To find $x^{1/c} = y$ using the right index on C: Move the cursor to 1.62 on LL2 and pull c on C under the cursor, then proceed. Examples; (1) $1.62^{1/6} = 1.0837$, set 6 on C and read 1.0837 on LL1 under the right index of C. (2) $1.62^{1/0.6} = 2.233$, set 0.6 on C and read 2.233 on LL2 under the right index of C. (3) $1.62^{1/0.06} = 3,040.$, set 0.06 on C and read 3,040 on LL3 under the right index of C. (Note: You only have to set the slide rule one time).

- **Quadratic equation:** Shown in the last pages of the Manual is the solution for the roots of a quadratic equation. The example equation given is: $2.16x^2 - 7.24x + 1.35 = 0$. To find the roots proceed as follows:

1. Divide by 2.16x and rearrange terms to obtain: $x + 1.35/2.16x = 7.24x/2.16x = 3.35$.
2. We take the second term $1.35/2.16x$ and call it y. So the root $y = 1.35/2.16x$.
3. Pick up the slide rule and set 2.16 on C to 1.35 on D. When we do that we find y will be on C1 and x on D. This gives us the two roots.
4. By trial and error we find y to be 0.198 and x to be 3.15 (this procedure is not fully explained in the manual).
5. Check $y = 0.198$: we have, $2.16 (0.198)^2 - 7.24 (0.198) + 1.35 = 0$
6. Check $x = 3.15$: we have, $2.16 (3.15)^2 - 7.24 (3.15) + 1.35 = 0$

- **Cubic equation:** Following this in the Manual is the general solution of the cubic equation $x^3 + px + q = 0$. This equation does not have the x^2 term so when divided by x it becomes the quadratic $x^2 + p + q/x = 0$, or when rewritten $x^2 + q/x = -p$. This is then solvable by the above method.

In the manual is a most unusual example using a slide that is reversed and turned upside-down. This is the first time I have seen this done. It looked like this:



This unusual configuration was used to solve for the root x in the cubic equation. The manual text is in Chinese and looks somewhat complicated. It was not able to be translated and so its explanation will not be included here.³

TRIG SCALES AND GAUGE MARKS

- **S** on the left and **(CS)** on the right. The S is the sine scale and the (CS) is the cosine. The scale runs from 5.74° to 90.0° . Set degrees on B and read value on C. Example: $\sin 22.3^\circ = \cosine 67.7^\circ = 0.379$.

Smaller values of the sine, from $1^\circ < \theta < 5.74^\circ$, will be found using the $\underline{T} 11 | 1 \underline{S}^\circ$ Mark at $57.3^\circ = 180^\circ / \pi$. For less than 1° the " and ' marks on the C scale are used. These are " at $206,265 = (180 \times 60 \times 60) / \pi$; and, ' at $3,438 = (180 \times 60) / \pi$. Examples: $\sin 3.25^\circ = 0.0567$. Set $\underline{T} 11 | 1 \underline{S}^\circ$ on C to 3.25 on D, then read 0.0567 on D under the right index of C. There are instructions in the manual for setting the cursor line into the spaces to the right of center to obtain more accuracy. For $\sin 18' = 0.00524$, set ' mark on C to 18 on D, read 0.00524 on D under the right index of C. For $\sin 35'' = 0.0001697$, set " mark on C to 35 on D, read 0.0001697 on D under the right index of C.

- **(T)** on the left and **CT** on the right. These are two separate major tangent scales. The (T) scale goes from $5.71^\circ \rightarrow 45^\circ$, and the CT from $45^\circ \rightarrow 84.29^\circ$.

The (T) scale is a tangent scale running from left to right. It decreases from $45^\circ \rightarrow 5.71^\circ$. It is used with the B, C, and D scales. Example: slide the left index on B to 33.4° above on T. Above the far right index of D read 0.659 on C. So, $\tan 33.4^\circ = 0.659$.

The CT scale (initials shown on the right side) is also a tangent scale. Running from left to right it increases from $45^\circ \rightarrow 84.29^\circ$. Example: slide the left index on B to 82.65° above on CT. The $\tan 82.65^\circ$ can be read on D below the right index of C as 7.75. So, $\tan 82.65^\circ = 7.75$

The tangent of small angles; $0.0^\circ \rightarrow 5.71^\circ$ are found using the Radian Mark $\underline{T} 11 | 1 \underline{S}^\circ$. Example: $\tan 3.25^\circ$. Slide 3.25 on D to the Radian Mark $\underline{T} 11 | 1 \underline{S}^\circ$. Slide the cursor to then right index mark of C. Read 0.0568 on D. So, $\tan 3.25^\circ = 0.0568$. There are instructions in the manual for setting the cursor line into the spaces to the left of center to obtain more accuracy.

ELEKTRO SCALES AND GAUGE MARKS

- **W:** On the (T) scale at about 16.8 there is an unusual gauge mark W, for Weight. Richard Smith Hughes refers to this mark in his Article, *Electro/Electro Slide Rules: A Critical Evaluation*.⁴ He says W is the weight from the formula $W = \gamma (L)(A) = \gamma (L)(\pi d^2 / 4)$, where γ is the specific gravity of copper wire ($\approx 8.9 \text{ cm}^3 / \text{gram}$. temperature insensitive), (L) is the total line length in yards, A is the wire's area, and d equals the wire diameter in mils. The gauge mark W is used to solve for W. Example: the Manual shows one problem using the mark W. I am guessing this is the way it goes. Under the mark W on scale (T) one sets (L) of 720 yds. on B. The hairline on the cursor is then moved to set A as 22.6 mils on D. The value of W (by the formula) is 3.34 lbs as read under the cursor line above on B.

- **R:** This R scale is for Resistance and stands alone on the right between the CU and (T) scales. The formula used by Richard Smith Hughes is $R \text{ (at } 15^\circ \text{ C)} = \rho (L) (\pi d^2 / 4) \text{ Ohm}$, where $\rho \text{ (at } 15^\circ \text{ C)} = \text{resitivity of Copper wire at } 15^\circ \text{ C} (\approx 8.9 \text{ cm}^3 / \text{gram}$. temperature sensitive). The values used are similar to the W example: A as 22.6 mils is set on D., and the cursor is moved over it. Then the slide is moved until 720 yds on B is under the cursor line. Move the cursor to the mark on the R scale and read 43.0 ohms on B. Once the resistance is found at the reference temperature (at 15° C), the resistance at any temperature may be found by moving the cursor line to the wanted temperature and reading the resistance on the B scale.

- **1 to 10** scale is in the space between the major B and S scales. It runs about halfway on the slide rule from left to right. This scale is used with the 30 | | 10 and 40 | 20 | 0 marks found between the D and LL2 major scales. The slide rule designer, *Mr. Jiang Dequ*, used the American Wire Gauge (AWG) wire designation. For example AWG wire (the wire used in the above example) has a listed diameter of 22.6 mils, the same as the slide rule answer. AWG 40 has a listed diameter of 331 mils and AWG 38 has a listed diameter of 4 mils. Example: For AWG 23, place cursor over the 20 mark, move the slide so that the 3 on the 1 – 10 scale falls under the cursor. Read 22.6 mils on D under the left index of C. This is the 22.6 mils used when completing the W example above.

HYPERBOLIC SCALES AND GAUGE MARKS

- **SH** scale is on the left and **TH** on the right. This is divided into two scales. The SH, hyperbolic sine, runs most of the scale left to right from 0.2 to 2.7. The TH, hyperbolic tangent, scale runs on the far right from 1 to ∞ . For obtaining values for SH, TH and CH special calculations are used. Some of these are found in a much different fashion than usually calculated with other slide rules that have hyperbolic function scales. Pay particular attention to the descriptions that follow (Remember, the hyperbolic functions are expressed in radians not degrees).

To find Sinh u values:

- **Sinh u for values smaller than Sinh 0.20** we can use the approximate formula $\text{Sinh } u \approx u$ (in radians). Example: $\text{sinh } 0.11 \approx 0.11$. Actual value is 0.110
- **Sinh u** values for u between $0.20 < u < 0.88$ are found using scale **Sh θ/θ** . This is shown on the far left of the SH scale. Example: for $u = 0.72$. Set the left index of C over 0.72 on D. Move cursor to 0.72 on Sh θ/θ and read 0.788 for Sinh u on the D scale under the cursor (actual value is 0.783840...).
- **Sinh u** values for u between $0.88 < u < 2.7$ are found using three different steps. These are shown in the following examples:

(1). Values between Sinh $0.88 < u < 1.00$: Example for value of sinh 0.9. Set right hand index of C to 0.9 on D. Move the cursor to 0.9 on the SH scale and read 1.03 on the D scale.

(2). Values between Sinh $1.00 < u < 2.22$: Example for value of sinh 1.55. Set left hand index of C to 1.55 on D. Move the cursor to 1.55 on the SH scale and read 2.25 on the C scale.

(3). Values between Sinh $2.22 < u < 2.7$: Example for value of sinh 2.45. Set right hand index of C to 2.45 on D. Move the cursor to 2.45 on the SH scale and read 5.75 on the C scale.

- **Sinh u** for values between $2.7 < u < 3.5$, are found using two different sets of special gauge marks. The gauge marks are found between scales CU and (T) and scales SH and C1.

For values between Sinh $2.7 < u < 3.0$: The special hyperbolic mark SH 11 | 11 CH 2.5 is used along with the ΔH scales to find sinh u.

Example: to find $\text{sinh } 2.98 = 9.82$. Set the cursor on the SH 11 | 11 CH 2.5 mark. Then move the slide so that 0.98 on the ΔH scale is under the cursor. Read $\text{sinh } 2.98 = 9.82$ on scale D under the right index of C. (Note that the scale ΔH runs from right to left from 0.0 to 1.0. The only values shown are 0.0 and 0.5. It starts on the right with 0.0 and runs left to 0.5. Then it starts again on the right with 0.5 and runs left to 1.0).

▪ **Sinh u** for values between $3.0 < u < 3.5$: The special hyperbolic mark SH 11 | 11 CH 3 is used along with the ΔH scales to find $\sinh u$. Example: to find $\sinh 3.45 = 15.73$, set the cursor on the SH 11 | 11 CH 3 mark. Then move the slide so that 0.45 on the ΔH scale is under the cursor. Read $\sinh 3.45 = 15.73$ on scale D under the left index of C.

▪ **Sinh u** for values larger than 3.5 . In the space between the major CU and (T) scales are a ten special **H** marks. The marks from left to right are as follows: SH 11 | 11 CH 2.5, H5, H.5, SH 11 | 11 CH 3, H1, H3.5, H1.5, H4, H2, H4.5. These various **H** marks are used to obtain higher values of $\sinh u$ from 3.5 and larger. We will explain how each mark is used.

For u values between $3.5 < u < 5.5$: This is done using one of the four of the ten H marks, the ΔH scale for decimal values of u , and then read Sinh u on the D scale below the C index:

- for Sinh $3.5 < u < 4.0$ use mark H3.5, and ΔH .
- for Sinh $4.0 < u < 4.5$ use mark H4, and ΔH .
- for Sinh $4.5 < u < 5.0$ use mark H4.5, and ΔH
- for Sinh $5.0 < u < 5.5$ use mark H5, and ΔH

Example: Sinh 4.8. Place cursor over the proper H scale on the body. This is H4.5. Move slide to bring decimal $u = 0.8$ on ΔH under cursor.

Read Sinh = 60.8 on the D scale below the scale C left index. (actual value is 60.7511...).

Example: Sinh 5.32. Place cursor over the proper H scale on the body. This is H5. Move slide to bring decimal $u = 0.32$ on ΔH under cursor.

Read Sinh = 102.2 on the D scale below the scale C left index. (actual value is 102.1895...)

For u values between $5.5 < u < 10.0$: This is done using one of the ten H marks, the ΔH scale for decimal values of u , and then read Sinh u on the D scale below the H5 gauge mark (this H5 gauge mark is found on the slide under the ΔH scale):

- for Sinh $5.5 < u < 6.0$ use mark H0.5, and ΔH ; then H5 and D.
- for Sinh $6.0 < u < 6.5$ use mark H1, and ΔH ; then H5 and D.
- for Sinh $6.5 < u < 7.0$ use mark H1.5, and ΔH ; then H5 and D.
- for Sinh $7.0 < u < 7.5$ use mark H2, and ΔH ; then H5 and D.
- for Sinh $7.5 < u < 8.0$ use mark H2.5, and ΔH ; then H5 and D (for the mark H2.5 use SH 11 | 11 CH 2.5).
- for Sinh $8.0 < u < 8.5$ use mark H3, and ΔH ; then H5 and D.
- for Sinh $8.5 < u < 9.0$ use mark H3.5, and ΔH ; then H5 and D.
- for Sinh $9.0 < u < 9.5$ use mark H4, and ΔH ; then H5 and D.
- for Sinh $9.5 < u < 10.0$ use mark H4.5, and ΔH ; then H5 and D.
- for Sinh $10.0 < u < 10.5$ use mark H5, and ΔH ; then H5 and D.

Example: Sinh 7.75. Place cursor over the proper H scale on the body.

This is H2.5. Move slide to bring decimal $u = 0.75$ on ΔH under cursor.

Read Sinh = 1160.0 on the D scale below the H5 gauge mark (actual value is 1160.786...).

Example: Sinh 9.12. Place cursor over the proper H scale on the body.

This is H4. Move slide to bring decimal $u = 0.12$ on ΔH under cursor.

Read Sinh = 4570.0 on the D scale below the H5 gauge mark (actual value is 4568.1007...).

For values between $10.0 < u < 15.5$: This is done using one of the ten H marks, the ΔH scale for decimal values of u . Then read Sinh u on the D scale below the H10 gauge mark (this H10 gauge mark is found on the slide to the right of the ΔH scale):

- for Sinh $10.5 < u < 11.0$ use mark H0.5, and ΔH ; then H10 and D.
- for Sinh $11.0 < u < 11.5$ use mark H1, and ΔH ; then H10 and D.
- for Sinh $11.5 < u < 12.0$ use mark H1.5, and ΔH ; then H10 and D.
- for Sinh $12.0 < u < 12.5$ use mark H2, and ΔH ; then H10 and D.
- for Sinh $12.5 < u < 13.0$ use mark H2.5, and ΔH ; then H10 and D.
- for Sinh $13.0 < u < 13.5$ use mark H3, and ΔH ; then H10 and D.
(for the mark H3 use SH 11 | 11 CH 3).
- for Sinh $13.5 < u < 14.0$ use mark H3.5, and ΔH ; then H10 and D.
- for Sinh $14.0 < u < 14.5$ use mark H4, and ΔH ; then H10 and D.
- for Sinh $14.5 < u < 15.0$ use mark H4.5, and ΔH ; then H10 and D.
- for Sinh $15.0 < u < 15.5$ use mark H5, and ΔH ; then H10 and D.

Example: Sinh 13.28. Place cursor over the proper H scale on the body. This is H3. Move slide to bring decimal $u = 0.28$ on ΔH under cursor. Read Sinh = 292,000 on the D scale below the H10 gauge mark (actual value is 292,685.1741...).

Example: Sinh 14.12. Place cursor over the proper H scale on the body. This is H4. Move slide to bring decimal $u = 0.12$ on ΔH under cursor. Read Sinh = 678,000 on the D scale below the H10 gauge mark (actual value is 677,966.2720...).

- **An alternative method for values larger than Sinh 3.5.** We can use the formula $\sinh u \approx e^u / 2$ for an approximate value. Example: to find $\sinh 9.12 \approx e^{9.12} / 2 = .$ Set cursor on 9.12 C and read 9120.0 on scale LL3. Then, $\sinh 9.12 \approx 9,120.0 / 2 = 4,560$. The actual value is 4,568.1008.

The above descriptions for obtaining values of **Sinh u** employed unusual procedures using many special scales and marks. The Author has not seen these methods used in any other slide rule and to his knowledge are unique.

To find Tanh u values: These are found on the **TH** scale that runs from $1.0 < u < \infty$ on the far right, and the **Th θ / θ** scale that runs from $0.0 < u < 1.0$ under the TH scale.

Example: $\tanh 1.4 = 0.885$, set the cursor on the right index of D, slide 1.4 on TH under the cursor, read 0.885 on C above the right index of D.

Th θ / θ : This scale is for smaller values $0.0 < u < 1.0$. Example: $\tanh 0.2 = 0.1974$, set the cursor on the right index of D, slide 0.2 on Th θ / θ under the cursor, read 0.1974 on C above 0.2 on D.

For practical purposes, for large $u > 3.0$ the $\tanh u \approx 1.0$.

To find Cosh u values: The $\cosh u = \sinh u / \tanh u$. Example: $\cosh 1.5 = \sinh 1.5 / \cosh 1.5$. Set cursor on the right index of D, slide 1.5 on TH under the cursor, slide the cursor to 1.5 on SH and read $\cosh = 2.35$ under cursor on D. For other cosh values, procedure may be to first find TH and then SH. It may take a number of steps to obtain cosh values.

Now completed is the explanation of this slide rule. It is my hope that you will agree that this is a most interesting and unusual slide rule. Also, the addition of special scales and

methods not normally found makes this a remarkable find. In fact, it is the opinion that this is the ultimate Vector slide rule.

I would like to thank Richard Smith Hughes for his extra assistance in completing this Article. He sent me a large amount of information regarding the different scales and a picture of his slide rule. He read the draft of my article and offered comments. Thank you, Richard.

References

1. William K. Robinson, *Slide Rules With Hyperbolic Functions*, Journal of the Oughtred Society, 14:1, 2005 This article, together with a list of about 130 slide rules, may be found in an expanded version at: www.hyperbolicsliderules.com. In the list see Number 51 for this slide rule. There is another Article by Richard Smith Hughes and William K. Robinson in JOS, 14:2, 2005, on Chinese slide rules. A listing of these Chinese slide rules is shown in the JOS Plus section of the Oughtred Society website www.oughtred.org. See Number 27 on the list for this slide rule.
2. The wording shown was found on a Chinese internet site and copied. Unfortunately the site could not be referenced here as it was lost by the computer.
3. This was in Chinese and the explanation of the use of the reversed and upside down slide was difficult to understand. I would like to have a complete explanation. I will be happy to send a copy of the Manual pages showing this upside down slide to anyone contacting me.
4. Richard Smith Hughes, *Electro/Electro Slide Rules: A Critical Evaluation*, Journal of the Oughtred Society, 18:1, 2007. This Article was sent to me for reference by Richard.