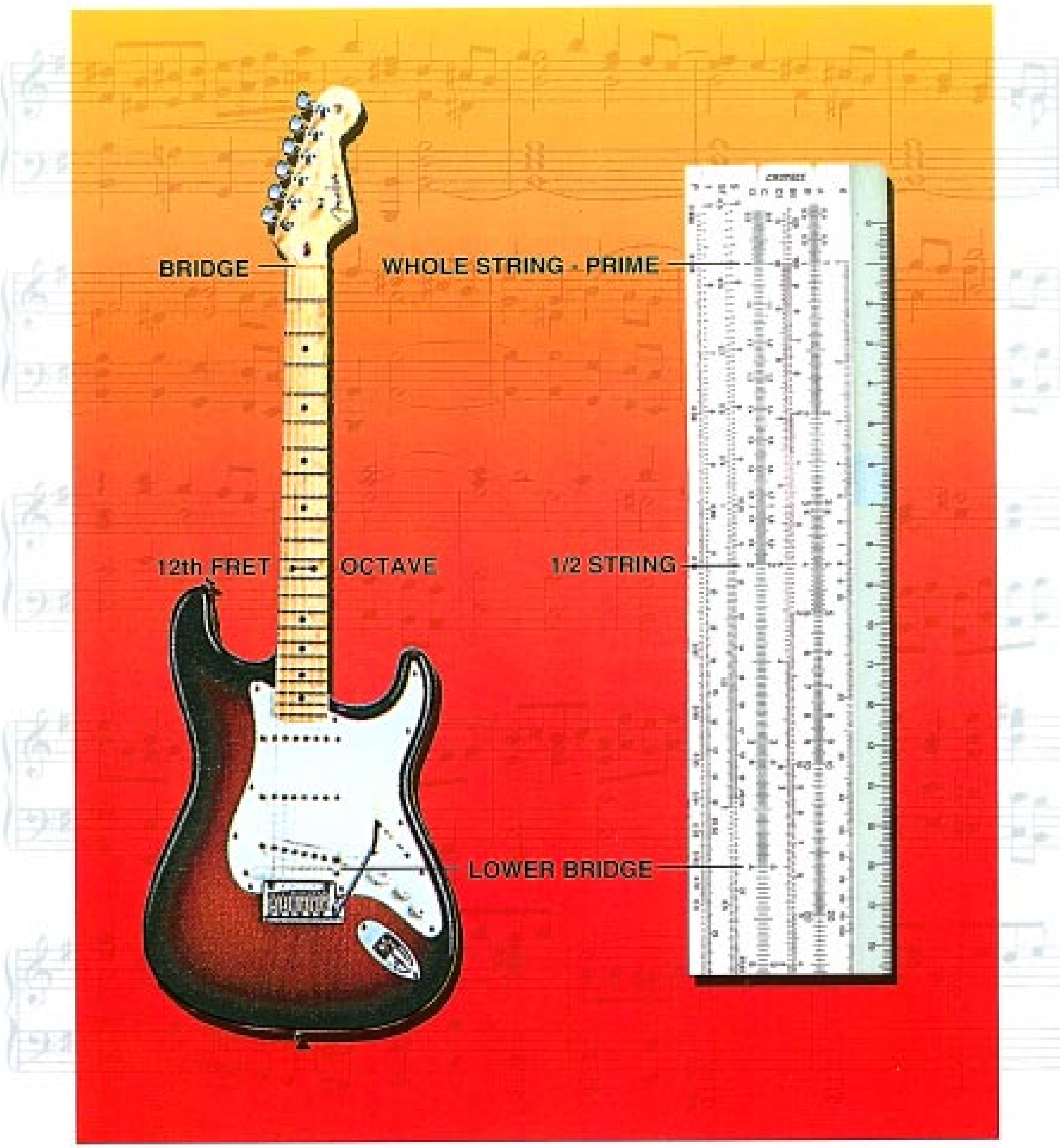
Calculating with Tones:
The Logarithmic Logic of Music



by Klaus Kuehn and Rodger Shepherd

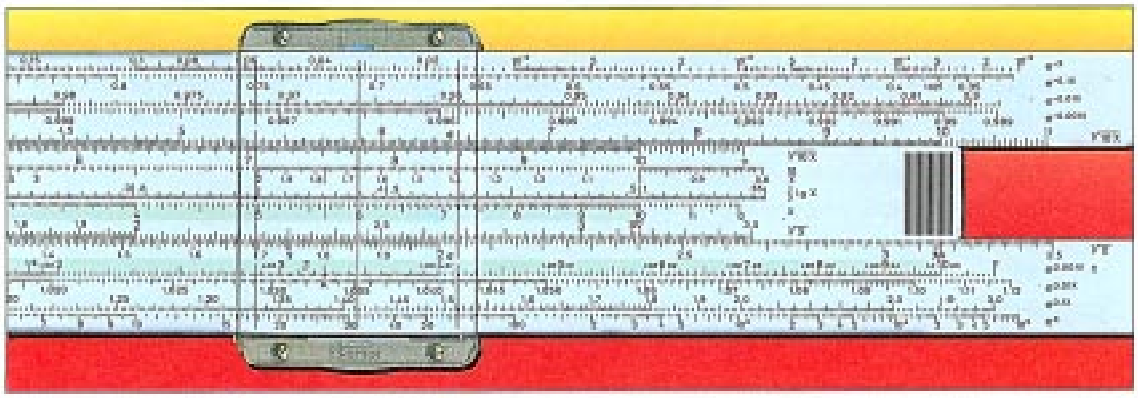


Figure 4.3

Settings for 600 Cents = $\sqrt[12]{2^6}$ =1.414 under x=6 on $e^{0.1x}$ on a Faber-Castell 2/83 N

Conversions into Cents can be carried out for all the scales (e.g., Pythagorian, pure diatonic, etc) described in this article. By doing such a conversion the distinctions between the different keys are easier to recognize. This is seen in the rounded numbers in Table 4.3. Distinct differences are apparent in the cases of the third, sixth, and seventh. These tone intervals were also those which were proposed in the formation of new tone intervals.

Table 4.3
Comparison of Cents in different pitches

Tone	Ratio of vibrations (based on C)	Dyadic logarithm	Cent	Ratio of vibrations (based on C)	Dyadic logarithm	Cent	Cent
	Pythagorian			Diatonic/Pure			Tempered
C	1	0	0	1	0	0	0
D	1.1250	0.1699	204	1.1250	0.1699	204	200
E	1.2656	0.3398	408	1.2500	0.322	386	400
F	1.3333	0.415	498	1.3333	0.415	498	500
G	1.5000	0.585	702	1.5000	0.585	702	700
A	1.6875	0.7549	906	1.6667	0.737	884	900
H	1.8984	0.9248	1110	1.8750	0.907	1088	1100
C'	2	1	1200	2	1	1200	1200

We can make a first approximation for the value of the "Pythagorian Comma" (Fifth Comma) based on Cent values for the pure and tempered fifth. Twelve fifths corresponds to 7 octaves, i.e., the following holds for the tempered scale:

$$12 \bullet 700 = 7 \bullet 1200 = 8400$$

The pure tone calculation is

$$12 \bullet 702 = 8424 \text{ -}$$

The difference of 24 Cents corresponds to the "Pythagorian Comma" (actually 23.46 Cents). This is the difference of ¼ of a semitone, a difference that can be noticed by a trained ear.

The "Third Comma" should be briefly considered here. The difference between the major and minor full tone can be expressed as follows:

$$\frac{9}{8} : \frac{10}{9} = \frac{9}{8} \bullet \frac{9}{10} = \frac{81}{80} = 1.01250$$

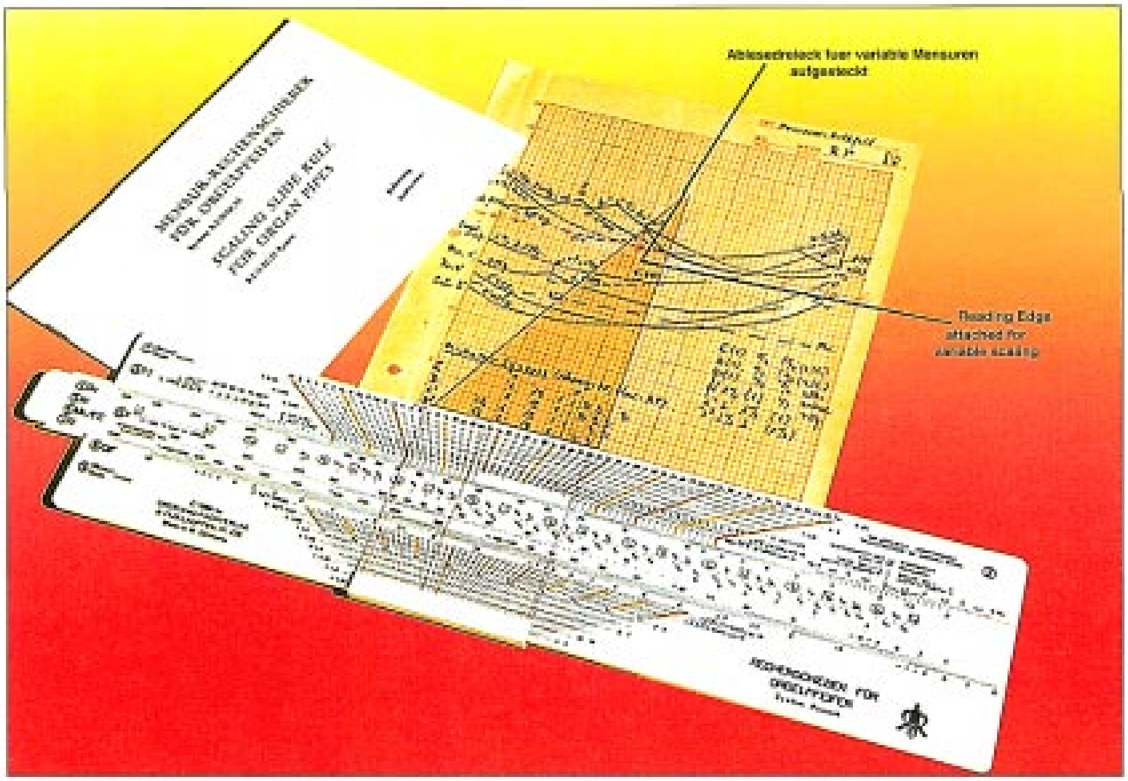


Figure 4.9
Photo of the scaling slide rule for organ pipes (System Rensch) as mentioned in the preface.

We have also included a copy of a flyer from Orgelbau Fachverlag 1986 (Fig. 4.10).

SCALING SLIDE RULE FOR ORGAN PIPES
Rensch System

The SCALING SLIDE RULE for ORGAN PIPES is indispensable for all organ-builders, pipe makers, organ case designers (pipe lengths!), organ restorers, consultants, organ experts, and organologists as well as for all serious organ fans. The Slide Rule makes an enormous simplification of designing pipe scales and their representation both as tables and graphs according to every imaginable system. All values such as diameters, plate width, half plate width, width of mouth according to every proportion between 3.5 and 8 for metal, conical values, end-compensation dependent on diameter and mouth-width are precisely indicated with only one adjustment per pipe. This is also valid for direct-reading of free-variable scales from a graph. The Slide Rule is also usable for all parts of reed stops such as width of resonators, length and diameter of shallots, tongue width etc.

The demand for the Scaling Slide Rule for Organ Pipes has continued in recent years, inspite of the development of micro-electronics and personal computers, so that a new edition of this reliable tool was necessary. Obviously the Scaling Slide Rule is much superior to any computer solution of pipe scaling not only because of its low price, but the device also supports the artistic and creative intuition which will never be possible with a computer. This is true in particular with free-variable scaling. Moreover, the Scaling Slide Rule is also irreplaceable for drawing up and analysing of extant pipes. All foot-pitches from 32' up to 1/32', including all applicable mutations and partials are specified on the Slide Rule. Constant pipe scale proportions from 1 : 1.25 (4 : 5) up to 1 : 2.1, or halving on the 16th, 17th etc. can be read directly without any further ado, including all values mentioned above. Special scales allow the correct conversion of wooden pipes into metal ones and vice versa, as well as the determination of change of pitch in Hz (cps) at temperatures between 0° and 30° Celsius (32° to 86° Fahrenheit).

On the back of the slide rule are tables of frequencies in Hz (cps), of the theoretical lengths based on a' = 435 Hz and a' = 440 Hz, as well as a millimeter scale. Twelve abridged instructions in symbols for the most-used functions of the slide rule are also there. A comprehensive hand-book with more than 30 practical examples comes with the slide rule, as well as 10 special transparent grid sheets (never protected, two scaling axes per sheet, more sheets available on request at any time). All inscriptions on the slide rule and the texts in the handbook are in English, too.

With the use of the Scaling Slide Rule for Organ Pipes, you will easily improve the tone quality of your organs by means of better pipe construction. If you have already spent much time on an individual scaling method, the use of the Scaling Slide Rule will, with your individual method, result in an enormous saving of time and increased precision. The risk of making mistakes will be reduced. The purchase cost is very small – the improvement of tonal quality will be great!

SCALING SLIDE RULE FOR ORGAN PIPES incl. accessories as mentioned above complete, in an elegant folding jacket — **U.S. $ 38.00** + $3.– S & H

To order, write to

Richard Rensch, c/o Jan Rowland, 16310 Cypress Garden Drive, HOUSTON, TX 77069
(Personal U.S.$ checks accepted – please add $2.00 redemption fee)

Figure 4.10
Brief description of the scaling slide rule for organ pipes

For figure 4.17, the upper image is by Arnold Schoenberg, Blaserquintett op.26, 1923-1924. Reihendrehscheiber, MS 26. Diameter 13cm. Arnold Schoenberg Center, Vienna. The lower image is by Arnold Schoenberg, Suite op. 29, 1925-1926. Reihenschieber, MS 29, 11 x 20.6 cm. Arnold Schoenberg Center, Vienna.

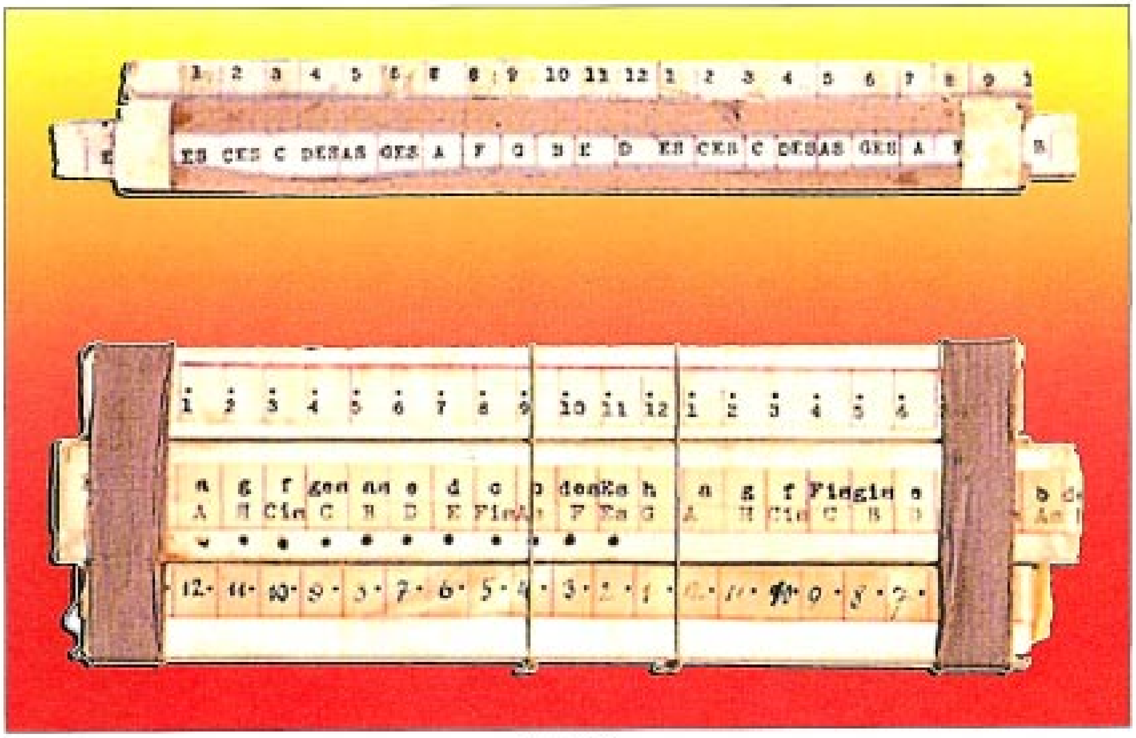


Figure 4.18
Twelve-tone selection slide rules

For figure 4.18, the above image is twelve-tone slide rule for the "Serenade, op. 24," by Arnold Schoenberg. The lower image is twelve-tone slide rule for the "Wind Quintet, op. 26," by Arnold Schoenberg. Arnold Schoenberg Center, Vienna.

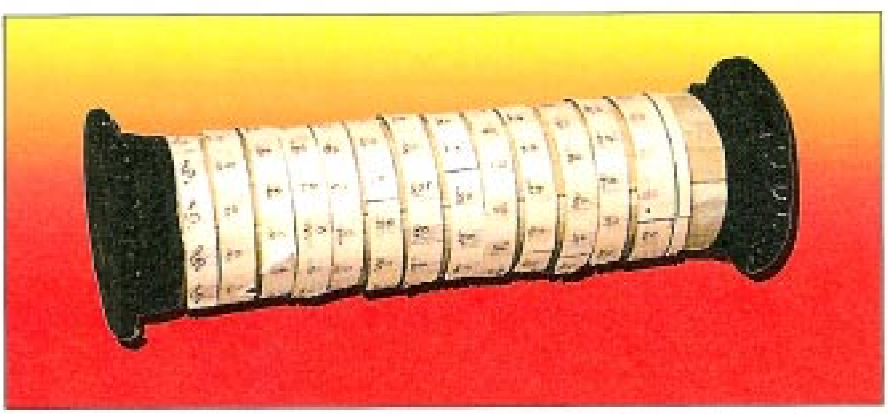


Figure 4.19
Twelve-tone row roll